Abstract

A high-frequency noise equivalent circuit for BJTs and HBTs is developed that extends established models and considers effects of shallow emitters with contacts of finite surface recombination velocity, drift fields in the base and signal delays due to finite collector transit time. Furthermore the noise due to carrier multiplication effects in the BC diode is included.

1. Introduction

Reliable BJT noise models are needed to accurately predict the noise performance of high-frequency circuits; a critical comparison of existing noise figure formulas has been presented recently [1]. Modern BJTs and HBTs are characterized by shallow emitters, base layers with strong drift fields and high collector doping densities. This work therefore investigates the consequences of emitter contacts with finite surface recombination velocity, high-frequency effects associated with a drift field in the base region and the collector signal delay, as well as consequences of carrier multiplication in the BC junction. It is shown how existing noise equivalent circuits have to be modified to consider these effects.

2. Transport Theory of Noise

Current and voltage fluctuations are due to generation and recombination processes or thermal motion of carriers and can be modeled by adding stochastic noise sources to the current and continuity equations [2, 3]. For the computation of noise generally a small-signal analysis is performed; the resulting system of equations may then be solved using Green’s function methods [4, 5]. In the quasi-neutral regions, the following equations apply

\[
\Delta J_n = e \mu_n E \Delta n + e \mu_n n_0 \Delta E + e \eta_n
\]

\[
\Delta J_p = e \mu_p E \Delta p + e \mu_p p_0 \Delta E - e \eta_p
\]

\[
- \frac{1}{e} \frac{\partial \Delta J_n}{\partial x} = - \frac{\Delta n}{\tau_n} - \frac{\partial}{\partial t} \Delta n + \gamma
\]

\[
- \frac{1}{e} \frac{\partial \Delta J_p}{\partial x} = \frac{\Delta p}{\tau_p} + \frac{\partial}{\partial t} \Delta p - \gamma
\]

if one-dimensional carrier transport in the x-direction is assumed. The terms \( \eta_n, \eta_p \) and \( \gamma \) denote stochastic noise sources, which describe microscopic fluctuations of carrier velocity and density, caused by thermal motion and generation-recombination processes. These sources are assumed to show no cross correlation, and to describe white noise represented by spectral density functions

\[
S_\gamma(x, x', \omega) = \frac{2 n_0^2 (x) + n_0 p_0}{A_j \tau_n} \delta(x - x') \quad (1)
\]

and \( S_{\eta p}(x, x', \omega) \) analogous

\[
S_{\eta p}(x, x', \omega) = \frac{4}{A_j} D n_0 p_0(x) \delta(x - x') \quad (2)
\]

The system of small-signal Langevin equations can be solved analytically making use of the adiabatic approximation [3], according to which the electron and hole densities in the space charge region are assumed to be in equilibrium with the n- and p-type region, respectively. According to this, fluctuations \( \Delta n(x, t) \) and \( \Delta p(x, t) \) of the mobile carrier densities are restored almost immediately within the space charge region and may therefore be neglected. This condition serves as a boundary condition for the solution of the Langevin equations.

![Figure 1. Noise equivalent circuit for the description of electron transport through the base region](image)

3. Equivalent Circuit Representation of Transfer Current Noise

NPN transistors are considered in the following. The small-signal (electron) transfer current passing the neutral base layer is described in terms of the admittance parameters \( y_{m0j} \) (underscores denote complex numbers). The noise associated with this current can be described in
terms of two correlated noise current sources as is illustrated in Fig. 1. The spectral densities of these noise current sources were calculated with the help of the stochastic transport equations; with \( n_p 0 \approx 0 \) in the base layer, the result

\[
\langle L_{ncn} L_{ncn}^* \rangle = 2 e I_T \Delta f
\]  

(3)

was obtained without any further approximation: The noise associated with the transfer current \( I_T \) thus follows the classical shot noise formula. If recombination in the base layer is neglected, the spectral density

\[
\langle L_{nbn} L_{nbn}^* \rangle = 4 k_B T \text{Re}(y_{n1le}) \Delta f
\]  

(4)

results in analogy with van der Ziel’s result [6] for the diffusion transistor, where \( \text{Re}(y) \) denotes the real part of \( y \). The input admittance \( y_{n1le} \) is given by

\[
y_{n1le} = g_m \left[ \frac{\vartheta}{\eta/2} \frac{\sinh(\eta/2)}{\sinh \vartheta} \left( e^{-\eta/2} \cosh \vartheta - 1 \right) + 1 \right]
\]

where \( \vartheta = \sqrt{(\eta/2)^2 - 2 \omega \tau_{BB0}, \tau_{BB0} = -E \delta B / V_T, \tau_{BB0} = \delta B / 2 D_n, g_m = I_T / V_T \) and \( \delta B \) denotes the width of the base layer. The physical origin of this noise current is the hole current required to neutralize the fluctuating number of electrons in the base region. The correlation between transfer current and base current noise is described by

\[
\langle L_{ncn} L_{nbn}^* \rangle = 2 k_B T \left( y_{n21e} - g_m \right) \Delta f ,
\]  

(5)

with

\[
y_{n21e} = g_m \frac{\vartheta}{\eta/2} \frac{\sinh(\eta/2)}{\sinh \vartheta} .
\]

The correlation between the noise sources increases with electric field strength, as expected. To illustrate the consequences of the improved description, the noise figure that results from the noise equivalent circuit Fig. 1 is compared with the result obtained from the corresponding noise equivalent circuit that is used in SPICE, shown within broken lines in Fig. 2. It was found that the deviation of the correct noise factor from the result of the quasistatic approximation may be represented as

\[
\Delta F = \Delta_1 + g_m R_S \Delta_2 + \frac{1}{g_m R_S} \Delta_3 ,
\]  

(6)

with functions \( \Delta_1(\omega), \Delta_2(\omega) \) and \( \Delta_3(\omega) \), which can be expressed in terms of \( y_{n1le} \) and \( y_{n21e} \). Fig. 3 shows a graphical representation of numerical results. As \( \Delta_2(\omega) \) has opposite sign than \( \Delta_1(\omega) \) and \( \Delta_3(\omega) \), the effects on the noise figure may compensate to a certain degree - dependent on the product \( g_m R_S \).

![Figure 2. Quasistatic noise equivalent circuit for the description of electron transport through the base](image)

![Figure 3. Correction terms \( \Delta_1(\omega), \Delta_2(\omega) \) and \( \Delta_3(\omega) \) vs. \( \omega^* = \omega \tau_{BB} \) for different values of \( \eta \)](image)

### 4. Shallow Junctions with Polysilicon Contact

In a forward-biased shallow junction with polysilicon contact (e.g. the emitter of a modern polysilicon-contacted npn BJT), the excess hole density at the contact will be different from zero. If the contact is characterized by the surface recombination velocity \( S_p \), the following relation between the small-signal hole current density and the small-signal excess hole density at the contact applies

\[
\Delta J_P(d) = -e D_p \frac{d}{dr} \Delta P \left|_{r_0} \right. = e S_p \Delta p(d) + e \xi .
\]  

(7)
The term \( \zeta \) denotes a stochastic term that takes account of fluctuations of the recombination at the surface and allows a consistent consideration of \( 1/f \) noise; neglecting low-frequency noise the spectral density

\[
S_n(\omega) = \frac{2}{A_1} S_p \left[ p_n(d) + p_{n0} \right]
\]

applies. With equation (7) used as a boundary condition, the stochastic transport equations were solved for a homogeneous n-type emitter with thickness \( d \) to obtain the spectral density of the injected hole noise current

\[
S_{ip} = \langle L_{np} L_{np}^* \rangle / \Delta f = 4kT Re(y_{np}) - 2eI_p,
\]

where

\[
y_{np} = \frac{e(I_p + I_{Sp}) (\theta_2(\omega) \theta_1(0))}{kT} \sqrt{1 + j\omega \tau_{ip}},
\]

with \( [L_{np}(\omega)] = I_p / \sqrt{1 + j\omega \tau_{ip}} \)

\[
\theta_1(\omega) = \sinh(d/L_{np}) + (D_{np}/S_p L_{np}) \cosh(d/L_{np}),
\]

\[
\theta_2(\omega) = \cosh(d/L_{np}) + (D_{np}/S_p L_{np}) \sinh(d/L_{np}),
\]

denotes the small-signal admittance due to hole injection into the n-type region; \( I_p \) denotes the hole current in the bias point, \( I_{Sp} \) the hole saturation current. If \( I_p \gg I_{Sp} \), the spectral density of the noise current can be written as \( S_{ip} = 2eI_p f_{p1} / \pi (\omega) \). Figure 4 shows \( f_{p1} (\omega) - 1 \) as a function of \( \omega \tau_{ip} \). Shallow polysilicon diodes thus show an increase of noise current with frequency that is more pronounced if the effective surface recombination velocity \( S_p \) becomes smaller (i.e. if \( \nu \) increases).

![Figure 4](image)

Figure 4. Plot of \( f_{p1} - 1 \) vs. \( \omega' = \omega \tau_{ip} \) for short diodes with \( d/L_{np} = 1/20 \) for different values of \( \nu = D_{np}/S_p I_{np} \).

5. High-frequency Noise Equivalent Circuit

With the correlated noise sources \( L_{eb} \) (4) and \( L_{ec} \) (3) to characterize noise due to the transfer current, and \( L_{np} \) (8) to characterize noise due to holes injected into the emitter, we are now able to give the full noise equivalent circuit of the bipolar transistor. Figure 5 shows the noise equivalent circuit for noise figure computations, including series resistances, depletion capacitances and finite collector transit time. \( L_{eb} = L_{ebn} + L_{ebp} \) describes base current noise associated with electrons and holes crossing the EB depletion layer, \( Y_n \) denotes the admittance matrix introduced in section 3. The admittance matrix \( Y' \) can be expressed as

\[
\begin{align*}
Y'_{11} &= y_{11} e + y_{p} + j\omega(c_{je} + c_{jc}) \\
Y'_{12} &= y_{12} e - j\omega c_{je} \\
Y'_{21} &= y_{21} e^* (\phi(\omega)) - j\omega c_{je} \\
Y'_{22} &= y_{22} e^* (\phi(\omega)) + j\omega c_{je}
\end{align*}
\]

where \( y_{np} \) denotes the small-signal admittance due to hole current injection in the emitter region, the \( y_{n,\alpha,\beta} \) denote the small-signal admittances associated with the electron transport through the base layer, and

\[
\phi(\omega) = \frac{\sin(\omega \tau_{je})}{\omega \tau_{je}} e^{-j\omega \tau_{je}} \approx e^{-j\omega \tau_{je}}
\]

describes the signal propagation across the BC depletion layer. In the equivalent circuit this term is represented as a delay line with delay \( \tau_{je} \approx d_{je} / 2 v_{s} \), where \( d_{je} \) denotes the width of the BC depletion layer and \( v_{s} \) is the saturation drift velocity for electrons. The noise sources in the noise equivalent circuit Fig. 5 are represented by phasors in the standard way; except \( L_{eb} \) and \( L_{ec} = L_{ecn} \) all noise sources stem from independent physical processes and are therefore uncorrelated.
6. Effects of Carrier Multiplication

Carrier multiplication in the base collector junction is a stochastic phenomenon and therefore contributes to the noise of the transistor. The conventional noise equivalent circuit of the bipolar transistor has to be extended to take this into account. From the theory of the avalanche photodiode it is known, that the noise of a reverse biased pn-junction, in which a primary current $I_T$ is injected, can be described by a noise current source with the spectral density $[7]$

$$ S_i = 2e m_n^2 \phi(m_n) I_T, \quad (10) $$

where $m_n$ denotes the average multiplication factor and $\phi(m_n)$ represents the increase of noise due to the stochastic carrier multiplication process; in case of pure electron injection we may approximate

$$ \phi(m_n) \approx 2 - 1/m_n \quad (11) $$

in the weak avalanche regime. For $m_n \rightarrow 1$ we have $\phi(m_n) \rightarrow 1$ and transfer current noise is represented by a noise source with spectral density $S_i \rightarrow 2e I_T \approx 2e I_C$, between the internal collector node $c'$ and the internal emitter node $e'$. Additional noise appears for $m_n > 1$ due to the noise of the generated hole current injected into the base region. Therefore an additional noise source between the internal collector node $c'$ and the internal base node $b'$ of the small-signal equivalent circuit has to be introduced as is shown in Fig.6.

![Figure 6. Noise equivalent circuit of bipolar transistor operated in the avalanche regime (series resistances neglected, quasistatic approximation)](image)

The noise due to carrier multiplication in the BC space charge layer is a superposition of two contributions. The first contribution describes the average multiplication of the statistically fluctuating transfer current; this part is represented by the controlled current source (represented by a square symbol for distinction) $(m_n - 1)L_{nc}$ and fully correlated with $L_{nc}$. The second term describes stochastic fluctuations of the multiplication process and is therefore independent from all the other noise sources. This part is represented by the noise current source $L_{nn}$, described by the phasor

$$ L_{nn} = \sqrt{2e I_T m_n^2 \phi(m_n) - 1}\Delta f \cdot e^{j\varphi_{nn}}. \quad (12) $$

The combined effect of the two noise sources yields the spectral density

$$ S_i = \frac{1}{\Delta f} \left( L_{nm}^2 + m_n L_{nc}^2 \right) = 2e m_n^2 \phi(m_n) I_T \quad (13) $$

in accordance with (10). Adding these noise sources to Fig. 5 gives for the increase of the noise factor caused by carrier multiplication effects

$$ \Delta F = \frac{g_m}{2R_S} \left\{ 2(m_n - 1) \text{Re}\left( \frac{H_{nc} H_{nm}^*}{H_{ld}} \right) \right. $$

$$ \left. + \left[ m_n^2 \phi(m_n) - 2m_n + 1 \right] \left| \frac{H_{nm}^2}{H_{ld}} \right|^2 \right\}, \quad (14) $$

where $H_{ld} = L_v/V_{ce}, H_{nc} = L_v/L_{nc}$ and $H_{nm} = L_v/L_{nm}$; this term reduces to zero for $m_n \rightarrow 1$. At $V_{ce} = BV_{CEO}$ an increase of $F$ by typically 0.1 (corresponding to an increase of the noise figure of about 0.4 dB) will result if the optimum source resistance in the absence of carrier multiplication is chosen.

7. Summary

A high-frequency noise model for BJTs and HBTs has been derived from the drift-diffusion model, extended by stochastic noise sources to Langevin equations. The model improves on established models by considering HF effects associated with emitter contacts of finite surface recombination velocity, effects of a drift field in the base region (as e.g. in graded-base SiGe HBTs) and finite BC depletion layer transit time as well as carrier multiplication effects.