Metal Rings as Quantum Bits

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Abstract

We propose a device that meets the physical and quantum mechanical conditions required for the operation of interacting quantum bits. Metal rings embedded in a solid state substrate by means of silicon processing technology are considered as the basic computing elements. We investigate different set-ups and concepts that are compatible with the topology of the metallic rings. Accessing the rings for in- and output signals as well as achieving the exchange and quantization of information are two essential requirements for a proper operation of an array of communicating quantum bits. In particular, the superposition of the quantum states characterizing the quantum bit array is primordial in order to run quantum computing algorithms. Furthermore, decoherence must be dealt with in a controllable fashion in order to read out the signals before they loose the signature of the quantum information. This work reports on the processing and testing of two device configurations.

1. Introduction

During the last decade, quantum computing has become a topic of ever growing interest, particularly in the light of cryptography applications that are based on Shor's algorithm [1] for factoring large integers. According to the latter, the factorization of an arbitrary integer N requires the application of a quantum parallelism to calculate the period of the sequence

$$f(x) = a^x \text{mod} N \ , \tag{1}$$

where x and a are integers and a and N are co-prime.
Problems of this kind and similar ones are practically insoluble for a classical computer in terms of computation time and memory loss. Some of these problems however can be solved by a quantum computer, provided they can be translated into proper algorithms. In classical computing, bits can be found in one of two possible states, denoted by $|1\rangle$ and $|0\rangle$, linked to physical states, e.g. charged or uncharged capacitor plates. In quantum mechanics, it is possible to bring the physical system underlying a bit, in an arbitrary linear combination of those two basic states, thereby substantially enlarging the information space. More generally, a two state system with basic states $|0\rangle$ and $|1\rangle$ can be found in a general state $|q\rangle$ as

$$\begin{aligned} |q\rangle \ &= \ \alpha\,|0\rangle \ + \ \beta\,|1\rangle \,, \\ |\alpha|^2 \ + \ |\beta|^2 \ &= \ 1 \ . \end{aligned} \tag{2}$$

The latter equation reflects the observation that the probabilities of having any mixture of the basic states always add up to one.
Moreover, a register of quantum bits (*q-bits*) can be in a entangled state of superposition during the computations, until the information is read out and the system collapses into a incoherent state. In other words, all q-bits are linked to each other during that time. For n q-bits, the register state Ψ can be described by a linear combination of 2^n q-bit basis vectors :

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} \alpha_i \,|i\rangle \ . \tag{3}$$

For the application of Shor's algorithm [1], a quantum computer would be exponentially faster than a classical one. In terms of applications, a very good security protocol based on the factorization into prime numbers could be cracked by a classical computer in maybe a few million years (if at all), while a quantum could accomplish such a task in about a year. Examples like this one have resulted in widespread interest in the field of quantum information and computation in the last decade [2]. The most successful q-bit array has been realized recently by a group of scientists in the US [3] and it executed Shor's algorithm to expand 15 into its prime factors 3 and 5, using NMR techniques. Implementations based on silicon technology are searched for as the future of this field lies in the production of solid state q-bits and registers. In general, every quantum computing project has to deal with the following issues :

- the ability to create a well-defined q-bit,
- the requirement of having proper I/O facilities, not affecting the quantum states before read-out,
- a reproducible preparation of the initial state,
- to achieve coherence times that are longer than computation times,

- the link between measurement results and quantum probabilities.

As far as applications are concerned, the q-bits need to be fabricated in a reproducible and scalable fashion in order to bring them on a chip, which is crucial for the peripheral circuitry. One vital ingredient is quantum error correction in order to deal with interferences and the lack of isolation of the system, leading to the destruction of quantum parallelism. The obstacles on the way to build a quantum computer are known and it seems to be merely a question of time to find solutions to all of them.

2. Microscopic Rings

We investigate the use of micrometer or nanometer sized rings to benefit from quantum effects. A semi-closed ring, connected to bonding pads can serve as an in- or output structure and thus enable us to access the system. Applying a time-dependent current signal to such a structure produces a changing magnetic field which can induce a current in a 'free' (isolated) closed ring, i.e. a *computing element*. Thus, we are exploiting Faraday's induction law to use the electromagnetic field as the information carrier in the system. The free ring can also induce a current in a neighboring ring and so on and in principle we can read out the signal again, as illustrated in Fig. 1.

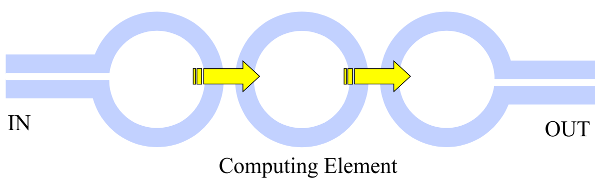

Figure 1. Basic Ring Concept

A major part of the work focuses on finding ways to both optimize the guidance of the magnetic flux between subsequent rings as well as to quantize the information carried by the flux transfer. A great deal of the magnetic field lines are found to spread out in space and therefore appropriate flux guiding has to be achieved to minimize or cancel information loss, thereby setting the classical basis for our device. We presently use ferromagnetic cores between the rings, similar to the case of a transformer. A soft permalloy with high permeability will switch its magnetic moments, according to the frequency of the driving signal, and continuously guide the flux from one ring to another. Signals in the range of a few µA up to 1 mA with frequencies below 10 MHz are sufficient to not exceed the saturation field of the permalloy and also to enable synchronized switching between the core and the magnetic field. We propose two ways for achieving quantization in the system, either by downscaling the ring sizes, or by using a superconducting material for the rings. The geometry of the isolated rings used for theoretical and numerical investigations is shown in Fig. 2.

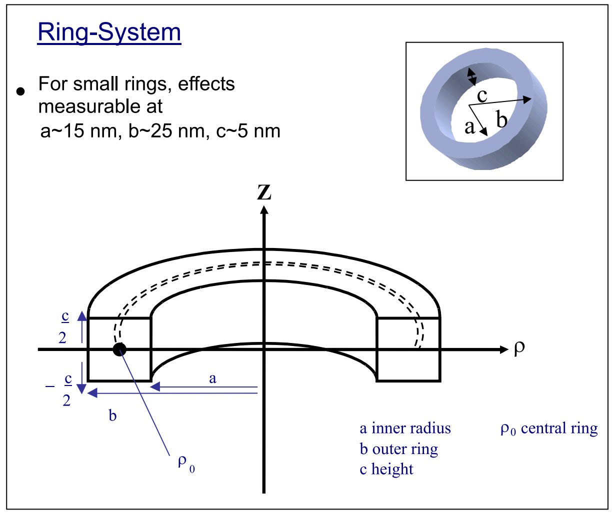

Figure 2. Ring geometry

For the case of *nanorings* (50 nm diameter), the electrons on the free ring are occupying discrete energy states and we may distinguish between the ground state and the first excited state and identify them as the basic $|0\rangle$ and $|1\rangle$ states. Assuming cylindrical symmetry, these eigenstates are also angular momentum eigenstates. The electron current density carried by a particular eigenstate $|n, m, p\rangle$ is approximately given by:

$$
\begin{aligned}
J_{n,m,p}(\rho, z) &= -\frac{2e\hbar}{\pi m_e (b-a) c \rho_0^2} \left(m + \frac{\Phi}{\Phi_0} \right) \\
&\quad \times \sin^2 k_n(\rho - a) \, \sin^2 k_p \left(z + \frac{c}{2} \right), \\
k_n &= \frac{n\pi}{b-a}, \; k_p = \frac{p\pi}{c}, \; \rho_0 = \frac{a+b}{2}, \\
m &= 0, \pm 1, \pm 2, \ldots \\
n, p &= 1, 2, 3, \ldots
\end{aligned} \tag{4}
$$

The inner and outer ring radii are denoted by a and b and the height by c. m_e represents the single-electron mass. The integers n and p are the quantum numbers associated with the motion in the radial and the z-direction respectively whereas m is the azimuthal quantum number. The constant $\Phi_0 = h/e$ is the elementary flux quantum while Φ is the total – external + induced – magnetic flux trapped by the inner circle $\rho = \rho_0, z = 0$ and coming in via the eigenenergies:

$$
E_{n,m,p} = \frac{\hbar^2}{2m_e} \left[k_n^2 + k_p^2 + \frac{1}{\rho_0^2} \left(m + \frac{\Phi}{\Phi_0} \right)^2 \right] . \tag{5}
$$

Next, we consider superconducting rings trapping multiples of the London flux quantum $\Phi_L = h/2e$ in the isolated ring. The logic states $|1\rangle$ and $|0\rangle$ now correspond to the presence or absence if a persistent current in the ring. This approach prevents the need for very small rings and simplifies the production of the first test structures.

Quantum mechanical simulations are needed to explain the measurement results and to enable traceability back to the quantization effects. In particular, it has to be figured out how the superposition of states may be detectable in an output signal.

3. The Solid-State Q-bit Design

Test structures are presently designed, consisting of basic aluminum ring arrangements (in- and output rings and computing element) as well as the ferromagnetic cores, made of nickel-iron (NiFe). Aluminum is a type I superconductor and the samples that are used in-house are found to be superconducting for temperatures below 1.23 K. NiFe has a relative permeability of about 75000. The layout is shown in Fig. 3. The structures are designed

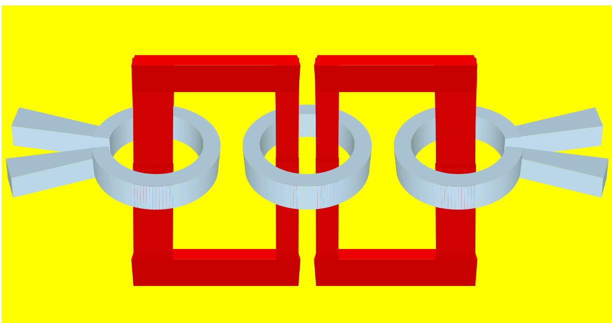

Figure 3. Device-Layout, Rings and Cores

in such a way that it is possible to perform electrical measurements as well as low temperature measurements using magnetic force microscopy (MFM). These experiments will enable us to verify a transformer-type of effect and also the flux quantization effect compatible with the presence of persistent currents in the ring. The above layout is realized, using lithography masks and standard processing techniques, such as deposition, etching and lift-off. In total, four device layers are present, embedded on a substrate:

- bottom core-parts,
- rings,
- core-tips,
- top core-parts.

4. Results

In the first stages of this work, efforts focuses on finding and testing appropriate flux guiding concepts as well as quantization options, each time being supported by simulations. The structures are drawn using the *Cadence Virtuoso* software. They include 15 µm diameter rings with thicknesses of 2 µm. The cores and tips are designed in such a way that all gaps, separations and minimal distances are 2 µm. Optical alignment structures and a passivations layer are included as well. Fig. 4 shows a typical part of the design, where three rings are present, two of them being connected to bonding pads. It also includes the other layers (cores, tips, passivation). The corresponding

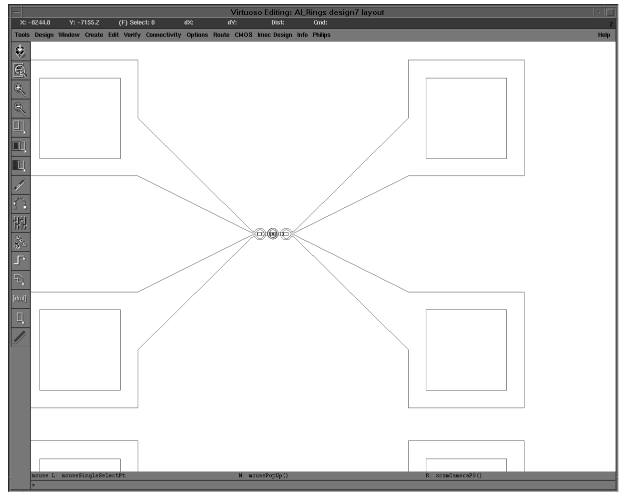

Figure 4. Design for three rings

lithography masks are made in-house. A process-flow is set up, using these masks to build the device on two-inch Si wafers in eleven processing steps. The crucial step is to connect the top and bottom parts of the cores by making trenches going through the rings, but not touching them. These are needed for the core-tips, to form closed structures.

Simulations [4] have supported the idea of using the ferromagnetic cores. 99.9 % of the flux can be guided from an input ring to a free ring, and 49.9 % of that flux can be guided to an output ring. Two cores share the available area on the free ring. There is sufficient coupling to get enough flux for creating a persistent current in a superconducting ring. The small cores enable us to use low current signals and still achieve relatively high magnetic fields (not exceeding the critical field strength). The following figure shows simulation results for three rings of 6 µm diameter and thicknesses of 1 µm, using NiFe cores of permeability 75000. An input current of 10 µA in the first ring produced fields up to 0.118 T inside the cores and was sufficient to achieve the desired coupling. We are also working on the derivation of the quantum mechanical equations describing the dynamics of the rings. They are being set up for both nanorings and the superconducting rings. Here, the aim is to create a suitable software tool in order to simulate the final ring system, both in terms of classical and quantum mechanical behavior, regardless of the type of quantization.

For superconducting rings, the supercurrent density may be derived from the time-independent *Ginzburg-Landau*

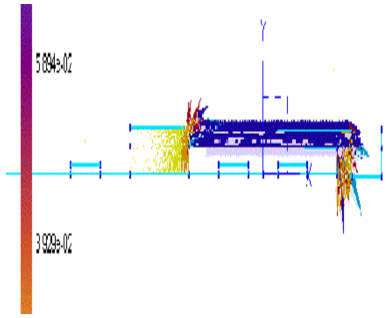

Figure 5. Effect of a magnetic core through the input and central ring (2D simulation)

equations,

$$\frac{1}{4m_e}\left(\mathrm{i}\hbar\nabla + 2e\mathbf{A}\right)^2\psi - \alpha\psi + \beta\left|\psi\right|^2\psi = 0\;. \quad (6)$$

It should be noted here that the electrons have paired up in reciprocal space to form *Cooper pairs*, which implies that the charge in this equation is $-2e$ whereas the effective mass of a Cooper pair is $2m_e$.
The total magnetic field $\mathbf{B}$ resulting from the induced field generated by the supercurrent and the fields arising from any other sources, comes in through the magnetic vector potential $\mathbf{A}$:

$$\mathbf{B} = \nabla\times\mathbf{A}\;. \quad (7)$$

The phenomenological parameters α and β are related to the density of Cooper pairs, $n_s = (\alpha/\beta)^{1/2}$, while α tends to zero near the critical temperature $T_c = 1.23K$ above which the normal state is completely restored. $\psi(\mathbf{r}) = \psi(\rho,\phi,z)$ denotes the Cooper pair wave function; it plays the role of a complex order parameter characterizing the superconducting state of the ring while its spatial distribution determines the supercurrent:

$$\mathbf{J}_\mathrm{s} = \frac{\mathrm{i}e\hbar}{2m_e}\left(\psi^*\nabla\psi - \left(\nabla\psi^*\right)\psi\right) - \frac{2e^2}{m_e}\mathbf{A}\;. \quad (8)$$

The detailed knowledge of the profile of $\mathbf{J}_\mathrm{s}$ as well as the number of flux quanta trapped by a particular ring for a given external field results from a self-consistent solution of the above-mentioned Ginzburg-Landau equations and the fourth Maxwell equation :

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J}_\mathrm{s}\;. \quad (9)$$

The dynamical equations for a full q-bit register still have to be worked out. They will not only involve the time-dependent Ginzburg-Landau equations and the corresponding time-dependent electromagnetic fields (including contributions from "normal" dissipative currents $\mathbf{J}_\mathrm{n}$) but also address the mutual interactions between neigboring rings in the register.

5. Conclusions

We proposed a method for implementing an array of solid-state q-bits which is innovative from both a quantum mechanical and a quantum computing point of view because the devices under investigation are not only intended to emulate a register of quantum bits but also contain extremely small transformers with permalloy cores to improve the flux guiding. Simulation results and measurements obtained from the test structures built up to date have given us sufficient information about the processing accuracy of the device as well as the physical behavior and exploitation. If we can back up our theoretical description with the planned measurements and finalized simulations, we will have a potential candidate for a solid state quantum computer.

6. References

[1] P. W. Shor, "Algorithms for quantum computation: Discrete logarithms and factoring", in Proc. 35th Annual Symposium on the Foundations of Computer Science, edited by S. Goldwasser (IEEE Computer Society Press, Los Alamitos, California), 1994, pp. 124-134.

[2] D. P. DiVincenzo, "Quantum Computation", Science **270**, 1995, pp. 255-261.

[3] L. Vandersypen et al., "Experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance (NMR) techniques", *Nature* **414**, 2001, pp. 883.

[4] C. Geuzaine, www.geuz.org, *GetDp/Gmsh software*