Abstract

A new method for extraction of small-signal equivalent circuit parameters has been developed. This method uses the entire frequency sweep in one calculation in opposite to other methods where one calculation is done for every measurement point. This makes it possible to extract more parameters than otherwise. The generality of the method makes it possible to extract the model parameters without any approximations or assumptions. This means that the model will be as accurate as the measurements and the choice of equivalent circuit. So far the method has been used only for extrinsic parameters but it can be extended to the complete circuit. To exemplify the technique, the small-signal equivalent model parameters for 0.18 μm RFIC MOS-transistors have been extracted.

1. Introduction

In the work with simulation models, the parameter extraction is of great importance. There are a number of problems associated with extractions. Most important is the de-embedding of pad parasitics, these parasitics includes everything between the calibration plane and the intrinsic device. Extensive research have been done in this field and a number of different methods have been published [1–3], which all have their pros and cons in terms of simplicity, accuracy and the type of devices they are suited for.

Extracting small-signal parameters involves a number of problems, which most of them are due to the limited number of data obtained from S-parameter measurements. From a 2-port measurement, eight independent parameters are obtained in each measurement point. The need for more complex models, due to the shrinking and more complex devices, increases the demand on the extraction technique. There exists several different strategies to extract complex small-signal circuits. A method originating from the ColdFET method used for MOS-FET assumes that the intrinsic device is purely capacitive when the applied biased are zero [4]. The series resistances are supposed to be independent of the bias and are extracted only when the biases are zero. Other existing methods take the intrinsic parts into account and extrapolate the curves toward infinite frequency to extract the series impedances [1, 5], which is done for each bias point. This is made possible by approximating the intrinsic part until it is simple enough.

In this paper we present a new technique to extract complicated small-signal circuits with emphasis on the extrinsic parts. The main difference between this method and other is the use of the entire frequency sweep in one calculation instead of making one calculation in each measurement point. The method extracts the parameter exactly. Exactly should be interpreted as no approximation and no assumptions except the choice of small-signal equivalent circuit. The method is general and can be used on a variety of circuit models, primarily FET models. We have chosen to extract only the series impedance in the first step even though it is possible to extract the entire cir-
cuit at once. The reason for that is to reduce the problems with complicated equations systems, which can be very demanding if the entire transistor is extracted at once. The series impedances can then be subtracted and the intrinsic part can be extracted in the same way or by some traditional extraction method depending on the circuit complexity.

2. Theory

For any small-signal equivalent circuit the analytical expression for the $Z$-parameters can be written as in (1-4). Note that the left hand side sometimes must be multiplied by $s$ so that $b_0 = 1$ and $a_0 \neq 0$ ($s = j\omega$, the complex frequency).

$$s_1 = \frac{a_{11}^1 + sa_{11}^1 + \cdots + s^{n_1}a_{n_1}^{11}}{1 + sb_1 + \cdots + s^{n_1}b_{n_1}} \quad (1)$$

$$s_2 = \frac{a_{12}^1 + sa_{12}^1 + \cdots + s^{n_1}a_{n_1}^{12}}{1 + sb_1 + \cdots + s^{n_2}b_{n_2}} \quad (2)$$

$$s_{21} = \frac{a_{21}^{11} + sa_{21}^{11} + \cdots + s^{n_1}a_{n_1}^{21}}{1 + sb_1 + \cdots + s^{n_2}b_{n_2}} \quad (3)$$

$$s_{22} = \frac{a_{22}^{12} + sa_{22}^{12} + \cdots + s^{n_2}a_{n_2}^{22}}{1 + sb_1 + \cdots + s^{n_2}b_{n_2}} \quad (4)$$

Each coefficient can be expressed in terms of the different parameters in the equivalent circuit. ($s^{n_1}a_{n_1}^{11}$ should be interpreted as $s$ to the power of $n_1$ times the coefficient $a_{n_1}^{11}$.)

By extraction of the coefficients using the least square method it is possible to obtain a equation system with more equations than unknowns. This makes it possible to extract several parameters without any approximations or assumptions. A slight problem is that the new equation system is overdetermined and thus care has to be taken on how to extract the parameters.

In order to calculate the $a$- and $b$-coefficients using the least square method the following matrix system (5) has to be evaluated:

$$X a = y \quad (5)$$

where $X$ is a ($p \times p$)-matrix, $a$ and $y$ are vectors with length $p$ where $p = n_{11} + n_{12} + n_{21} + n_{22} + m$. The matrices can be expressed using sub-matrices as in (6)-(8).

$$X = \begin{bmatrix} X_{a,11} & 0 & 0 & 0 & X_{c,11} \\ 0 & X_{a,12} & 0 & 0 & X_{c,12} \\ 0 & 0 & X_{a,21} & 0 & X_{c,21} \\ 0 & 0 & 0 & X_{a,22} & X_{c,22} \\ X_{c,11} & X_{c,12} & X_{c,21} & X_{c,22} & X_B \end{bmatrix} \quad (6)$$

$$a = [a_{11} \ a_{12} \ a_{21} \ a_{22} \ a_B]^T \quad (7)$$

$$y = [y_{11} \ y_{12} \ y_{21} \ y_{22} \ y_B]^T \quad (8)$$

The sub-matrices are thoroughly discussed in paper [6].

3. Results

The extraction method is applied on a 0.18 $\mu$m MOS-transistor ($W_C=90 \mu$m). The measured $S$-parameters were de-embedded using an open structure. To describe the transistor the small-signal equivalent circuit in fig. 1 was found sufficient in order to demonstrate this method. The analytical $Z$-parameter expressions for the small-signal scheme gives that $n_{11} = n_{21} = 4, n_{12} = n_{22} = 3$ and $m = 2$. When the order of the expressions are obtained a least square fit to the measurements was done, fig. 2. This gives the value of the terms in (9-11) and from the highest order terms the series inductances can easily be obtained.

$$L_g + L_s = a_{11}^{11}/b_2 \quad (9)$$

$$L_s = a_{12}^{12}/b_2 = a_{21}^{21}/b_2 \quad (10)$$

$$L_d + L_s = a_{22}^{22}/b_2 \quad (11)$$

Notice that $a_{12}^{12} = a_{21}^{21}$, if not, which probably is the case, the mean value of them should be used instead. This can be done directly in the least square matrix. The simplicity of the inductance extraction is independent of the equivalent circuit. Calculating the series inductances yields that $L_g=93 \, \text{pH}$, $L_s=62 \, \text{pH}$ and $L_d=25 \, \text{pH}$.

The analytical expressions are rewritten as in (12-16) and a new set coefficient is obtained.

$$z_{12} = \frac{a_{12}^{12} + a_{12}^{12} s + a_{12}^{12} s^2 + a_{12}^{12} s^3}{1 + b_1 s + b_2 s^2} \quad (12)$$

$$a_{12}^{12} s + a_{12}^{12} s^2 + a_{12}^{12} s^3 \quad (10)$$

$$a_{21}^{21} s + a_{21}^{21} s^2 + a_{21}^{21} s^3 \quad (11)$$

$$a_{3} = a_3 - L_s b_2 = 0 \quad (13)$$

$$a_{4} = a_2 - L_s \quad (14)$$

$$a_{5} = a_1 \quad (15)$$

$$a_{6} = a_0 \quad (16)$$

From the new set of coefficient the series resistances are calculated. This is done in a manner similar to the
inductance calculation. The series resistance extraction yields: $R_s=11 \, \Omega$, $R_d=16 \, \Omega$ and $R_i=2.2 \, \Omega$. From the analytical $Z$-parameter expressions the series impedances are removed by subtraction, resulting in the $Z$-matrix for the intrinsic part. The intrinsic parameters can then be extracted using the same method or as in this case, when the circuit is rather simple, by conventional direct extraction [1, 4, 5]. The result from the direct extraction for $V_C=0.8 \, \text{V}$ and $V_D=1.5 \, \text{V}$ are shown in table 1.

Parameter extraction has been done for several different biases all with good consistency. Fig. 3 shows the measured and the modeled $S$-parameters ($V_C=0.8 \, \text{V}$, $V_D=1.5 \, \text{V}$).

4. Discussion

This extraction method is able to calculate the series impedances with high accuracy in a simple manner. In addition the method does not rely on any approximation or assumption about the intrinsic part to be able calculate the series impedance.

If the intrinsic part of the equivalent circuit is complicated normal extraction method will not be useful. The proposed technique can then be used for this part too; due to the possibility to calculate a larger number of parameters. This can be done in different manners either the intrinsic parameters and series impedance parameters are calculated at once or divided in two calculations, where the first deals with the series impedance and the second with the intrinsic part. Using the method in the one step calculation tends to give a very complicate equation system, but handled in an appropriate way, once the calculation is done it will be a very powerful extraction tool. If the equation system is overdetermined some caution has to be taken in order to solve it. It should also be pointed
out that unlike optimization methods this technique does not require any start values, which make this a direct extraction method.

Normally the transconductance parameter in a small-signal equivalent circuit is expressed with a phase parameter \( \tau \) \( (g_m = g_{m0}e^{-j\omega \tau}) \). The proposed technique does not handle the sinusoidal functions that \( \tau \) introduces. If however \( \tau \) is needed it can be introduced by series expansion of \( e^{-j\omega \tau} \) with enough terms, which is fully covered by this method.

There are several ways of improving this method. Measurement data can be weighted according to uncertainty of the measurement equipment. Parameters where the significance changes with frequency could be weighted, for example inductances have significance at high frequencies.

5. Conclusion

We have presented a new general method for extracting small-signal equivalent circuits with emphasis on the series impedance. The technique is based on the least square method and utilizes the entire frequency sweep in one calculation, and therefore more complicated equivalent circuit models can be extracted. The strength of the method is that no approximations or assumptions have to be made.

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